THIRUTHANGAL NADAR COLLEGE

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Selavayal, Chennai-51.

A Self-Financing Co-educational College of Arts & Science

Affiliated to the University of Madras

Accredited with 'B' Grade by NAAC

An ISO 9001: 2015 Certified Institution

NAME OF THE DEPARTMENT : MATHEMATICS

SUBJECT : GRAPH THEORY

TOPIC : TYPES OF GRAPH

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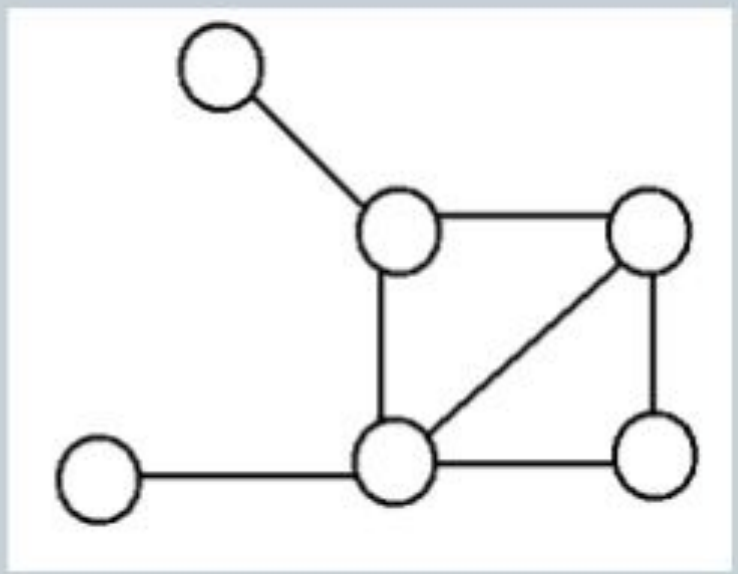
Types of Graph

Simple Graph (Undirected)

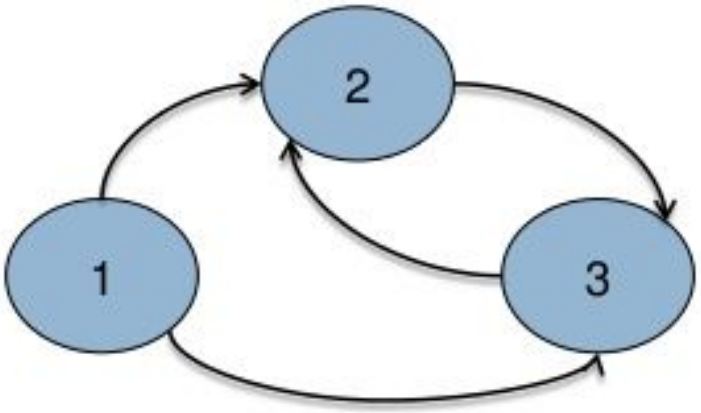
- Simple Graph are undirected graphs without loop or multiple edges
- A = AT



For simple graphs, $\sum_{v_i \in V} \deg(v_i) = 2|E|$

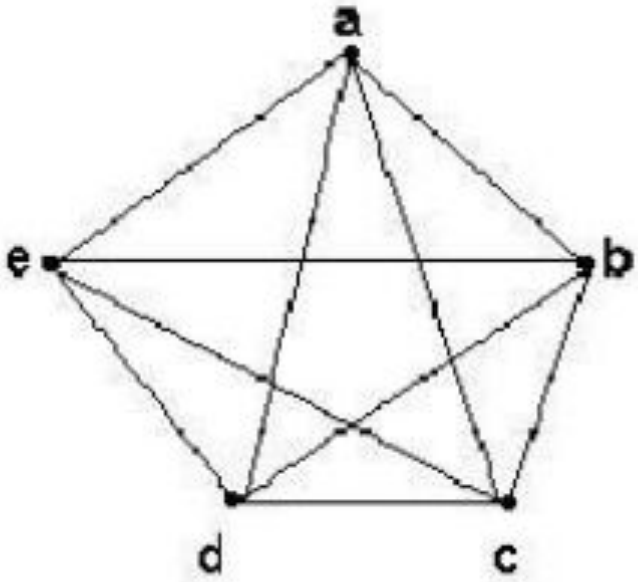
MultiGraph(Without Self Edge)

- The term multigraph refers to a graph in which multiple edges between vertices are permitted.
- A multigraph G = (V, E) is a graph which has the set of vetrices and multiple edges between vertices.



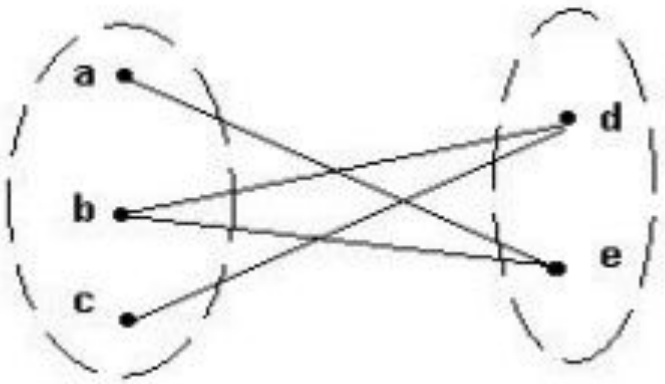
Complete graph K_n

- Let $n \geq 3$
- The *complete graph* K_n is the graph with n vertices and every pair of vertices is joined by an edge.
- The figure represents K_5

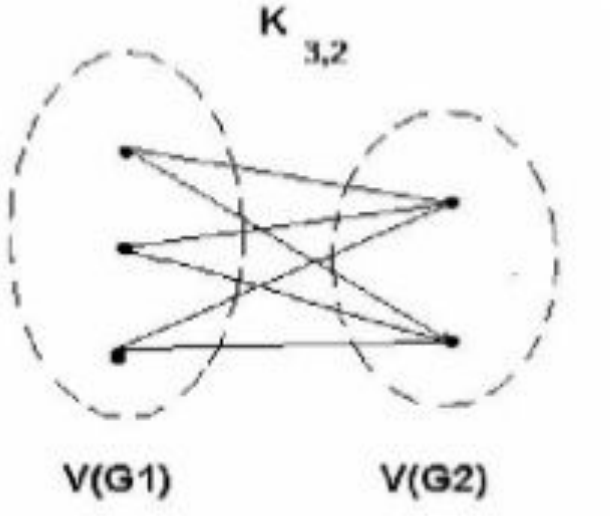


Bipartite graphs

- A bipartite graph G is a graph such that
  - $V(G) = V(G_1) \cup V(G_2)$
  - $|V(G_1)| = m$, $|V(G_2)| = n$
  - $V(G_1) \cap V(G_2) = \varnothing$
  - No edges exist between any two vertices in the same subset $V(G_k)$, k = 1,2



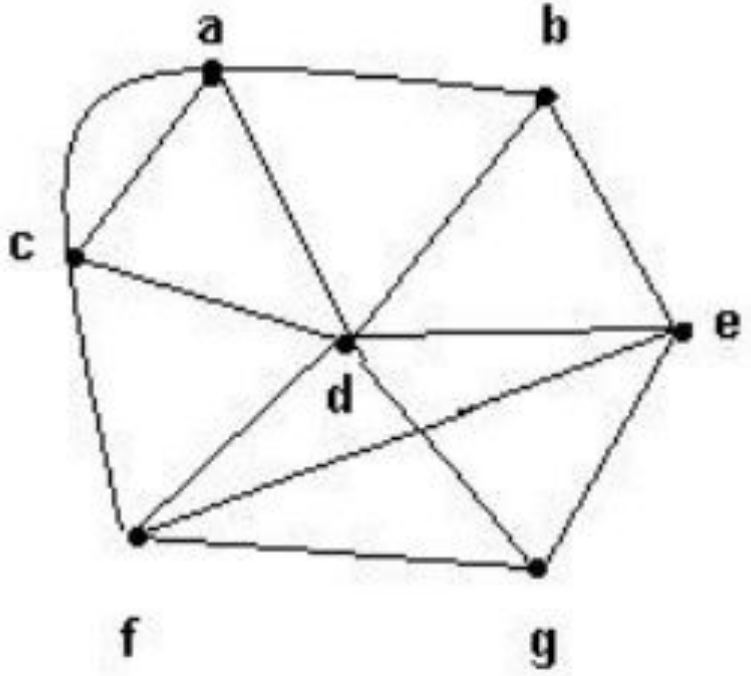
Complete bipartite graph $K_{m,n}$



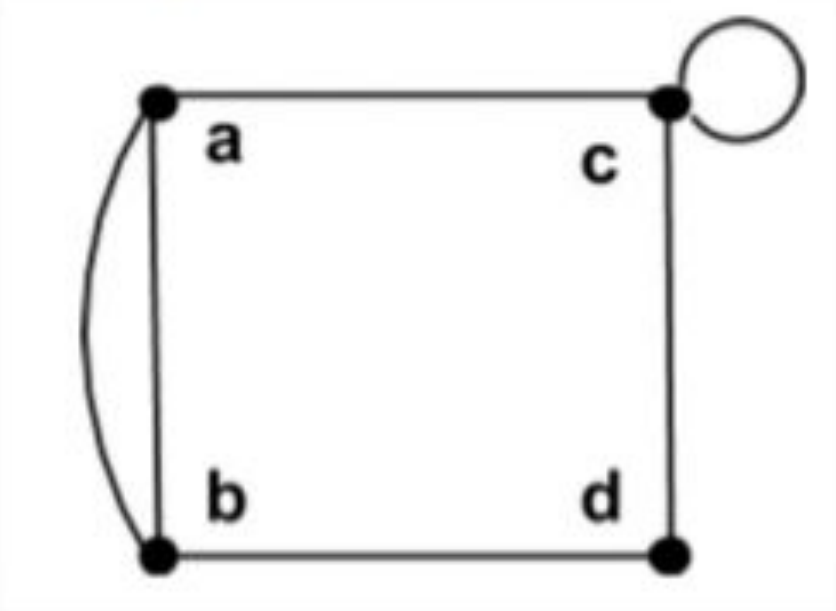
- A bipartite graph is the *complete* bipartite graph $K_{m,n}$ if every vertex in $V(G_1)$ is joined to a vertex in $V(G_2)$ and conversely,
- $|V(G_1)| = m$
- $|V(G_2)| = n$

Degree of a vertex

- The *degree* of a vertex v, denoted by $\delta(v)$, is the number of edges incident on v
- Example:
  - $\delta(a) = 4$, $\delta(b) = 3$,
  - $\delta(c) = 4$, $\delta(d) = 6$,
  - $\delta(e) = 4$, $\delta(f) = 4$,
  - $\delta(g) = 3$.



Parallel Edges / Self Loop



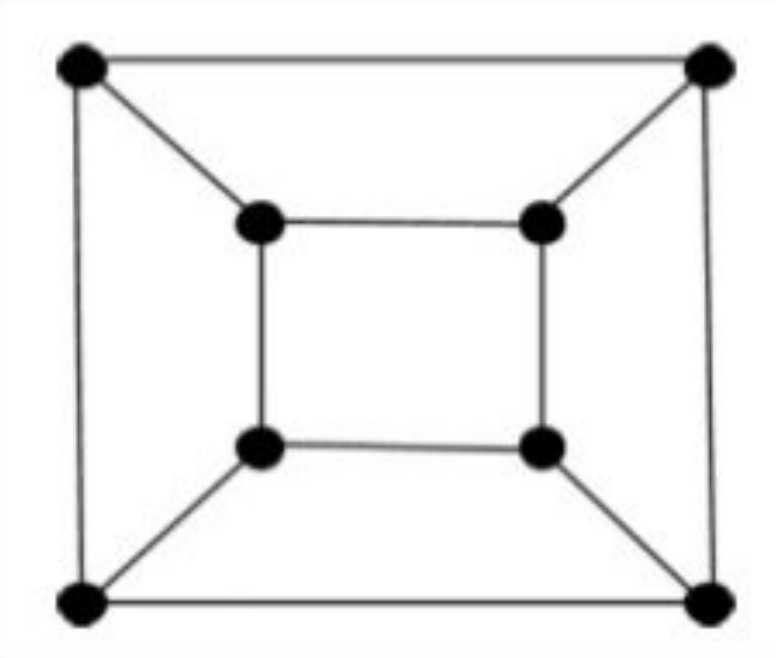
- Parallel Edges:
  - Multiple edges between the same pair of vertices
- Self Loop:
  - Edge between a vertex and itself

Regular Graph

- All the vertices have the same degree.
- ***D*-regular graph**:
  - All the vertices have degree *d*.
- [Question] Draw a 3-regular undirected graph with 8 vertices.

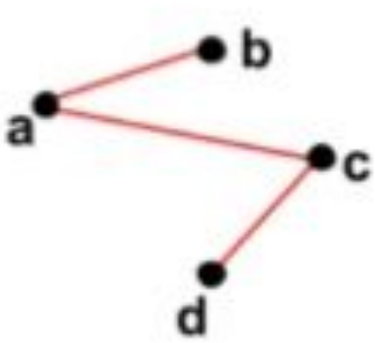
Regular Graph

- 3-regular graph

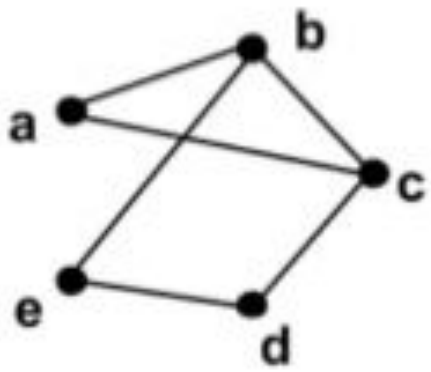


Subgraph

- G' = (V', E') is a subgraph of a graph G = (V, E) if
  - V' ⊆ V and E' ⊆ E



H

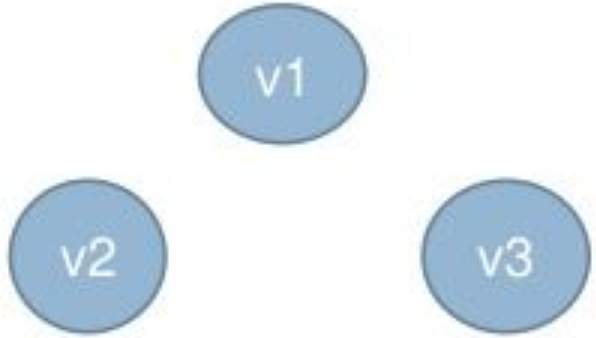


G

H is a *subgraph* of G

Null graph, Trivial Graph

- A graph G=(V,E) where E=0 is said to be Null or Empty graph

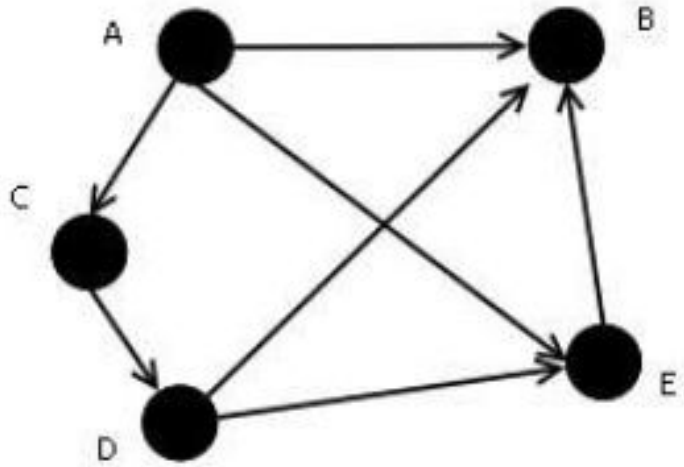


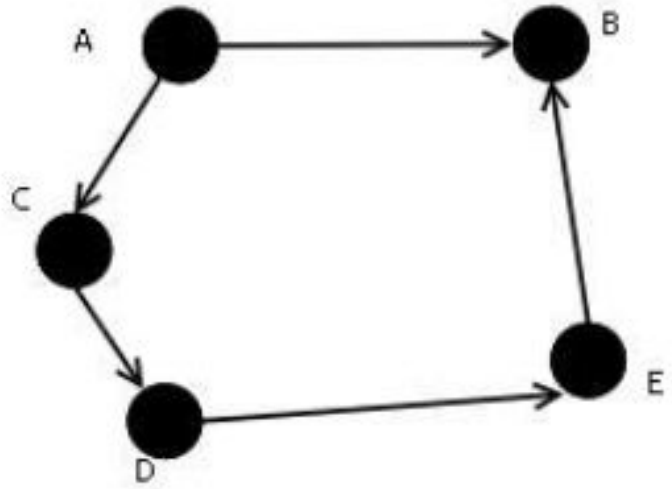
- A graph with One vertex

and no edge is called as a trivial graph.

Spanning Subgraph

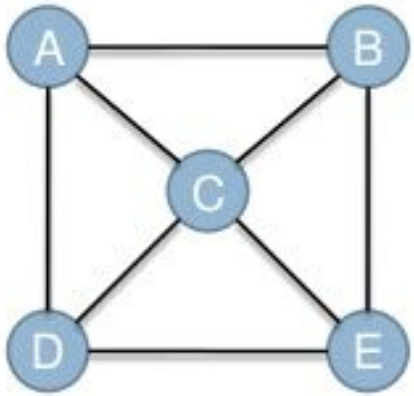
- A *spanning subgraph* is a subgraph that contains all the vertices of the original graph.

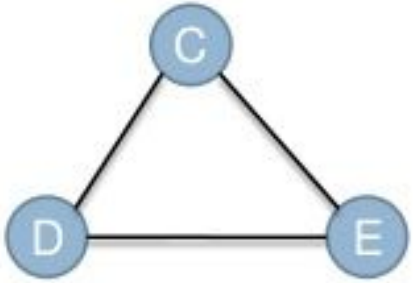


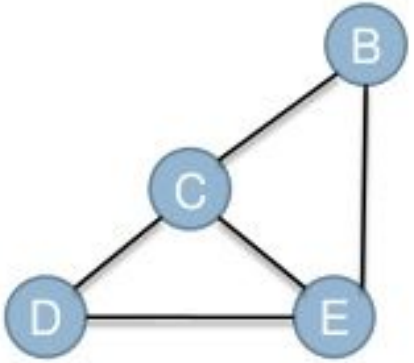


Induced-Subgraph

- Vertex-Induced Subgraph:
  - A *vertex-induced subgraph* is one that consists of some of the vertices of the original graph and all of the edges that connect them in the original.

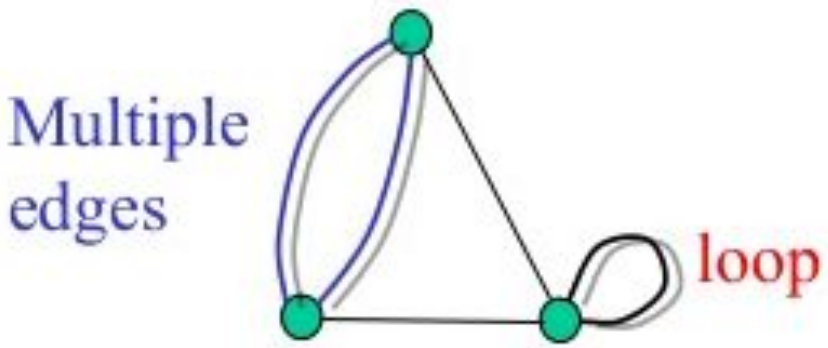






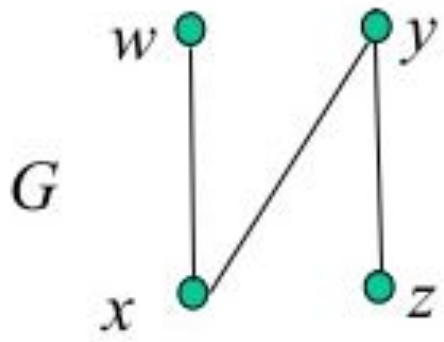
Loop, Multiple edges

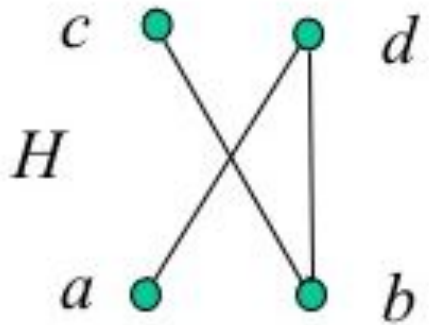
- *Loop* : An edge whose endpoints are equal
- *Multiple edges* : Edges have the same pair of endpoints



Isomorphism

- An ***isomorphism*** from a simple graph G to a simple graph H is a bijection $f: V(G) \rightarrow V(H)$ such that $uv \in E(G)$ if and only if $f(u)f(v) \in E(H)$
  - We say "***G is isomorphic to H***", written $G \cong H$





f_1: w x y z
c b d a

f_2: w x y z
a d b c


Thank
You