THIRUTHANGAL NADAR COLLEGE

(Belongs to the Chennaivazh Thiruthangal Hindu Nadar Uravinmurai Dharma Fund)

Selavayal, Chennai-51.

A Self-Financing Co-educational College of Arts & Science

Affiliated to the University of Madras

Accredited with 'B' Grade by NAAC

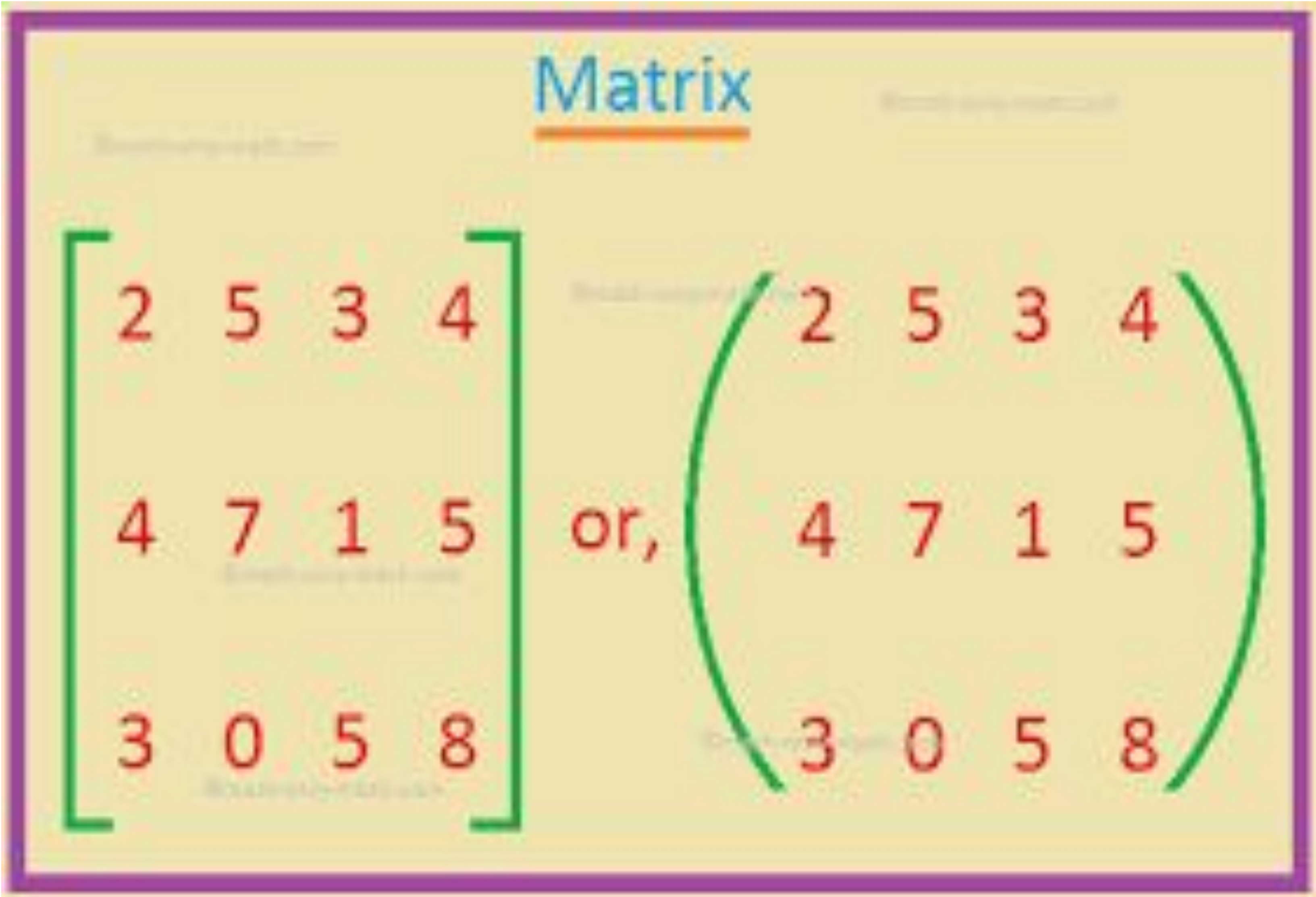
An ISO 9001: 2015 Certified Institution

NAME OF THE DEPARTMENT : MATHEMATICS

SUBJECT : MATHEMATICS-I(ALLIED)

TOPIC : MATRICES

STAFF NAME : CHITRA N


Matrix
2 5 3 4
4 7 1 5
3 0 5 8
or,
2 5 3 4
4 7 1 5
3 0 5 8

MATRICES:

$$A = \begin{bmatrix} a11 & a12 & a13 \\ a21 & a22 & a23 \\ a31 & a32 & a33 \end{bmatrix}$$

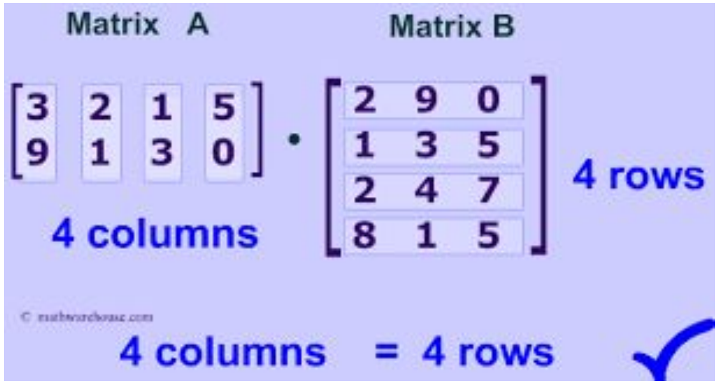
- **TOPICS:**
- **INTRODUCTION**
- **TERMS USED IN MATRICES**
- **TYPES OF MATRICES**
- **TRANSPOSE AND ITS PROPERTIES**
- **ADJOINT AND INVERSE OF A MATRIX**
- **SYMMETRIC AND SKEW-SYMMETRIC MATRICES**
- **HERMITIAN AND SKEW-HERMITIAN MATRICES**

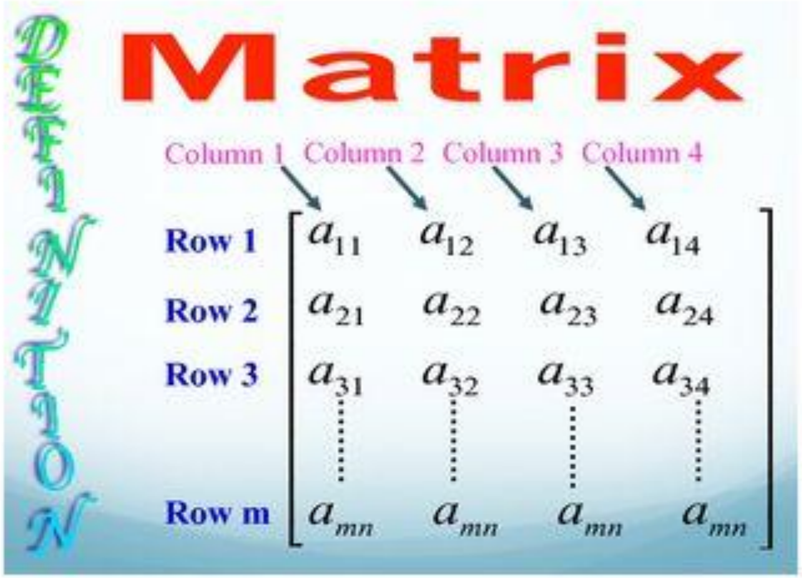
INTRODUCTION:

$$
\begin{array}{c} \\ 1 \\ 2 \\ 3 \\ \vdots \\ m \end{array}
\begin{array}{c}
\begin{array}{cccc} 1 & 2 & \cdots & n \end{array} \\
\begin{bmatrix}
a_{11} & a_{12} & \cdots & a_{1n} \\
a_{21} & a_{22} & \cdots & a_{2n} \\
a_{31} & a_{32} & \cdots & a_{3n} \\
\vdots & \vdots & \vdots & \vdots \\
a_{m1} & a_{m2} & \cdots & a_{mn}
\end{bmatrix}
\end{array}
$$

- A matrix (plural matrices) is a rectangular *array* or *table* of numbers, symbols, or expressions, arranged in *rows* and *columns*.
- An $m \times n$ matrix: the m rows are horizontal and the n columns are vertical. Each element of a matrix is often denoted by a variable with two subscripts. For example, $a_{2,1}$ represents the element at the second row and first column of the matrix.

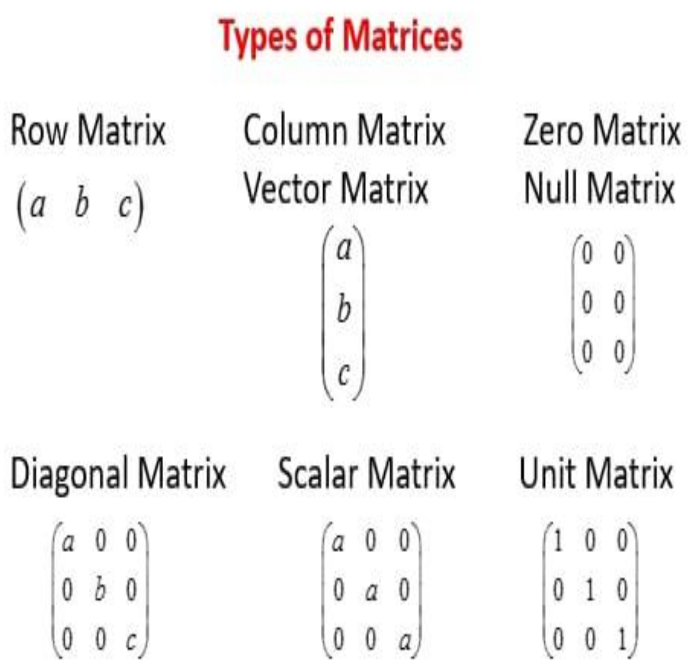
TERMS USED IN MATRICES:





- Element: An individual item in a matrix.
- Rows: The horizontal line of entries in a matrix
- Columns: The vertical line of entries in a matrix

TYPES OF MATRICES:



- Row matrix: A matrix with a single row.
- Column matrix: A matrix with a single column.
- Square matrix: A matrix which has the same number of rows and columns.
- Identity matrix: An Identity Matrix has 1s on the main diagonal and 0s everywhere.
- Diagonal matrix: A diagonal matrix has zero anywhere not on the main diagonal.
- Scalar matrix: A scalar matrix has all main diagonal entries the same, with zero everywhere else.
- Null matrix: A matrix whose all elements are zero.

TRANSPOSE of a matrix

- The transpose of a matrix can be defined as an operator which can switch the rows and column indices of a matrix i.e. it flips a matrix over its diagonal.
- **Properties of Transpose of a Matrix**

(i) Transpose of the Transpose Matrix

- If we take transpose of transpose matrix, the matrix obtained is equal to the original matrix. Hence, for a matrix A,
- (A′)′ = A
- What basically happens, is that any element of A, i.e. aij gets converted to aji if transpose of A is taken. So, taking transpose again, it gets converted to aij, which was the original matrix A.

(ii) Addition Property of Transpose

- Transpose of an addition of two matrices A and B obtained will be exactly equal to the sum of transpose of individual matrix A and B.
- This means,
- (A+B)′ = A′+B′

Types of Matrices

Transpose Matrix:

For a matrix $A_{m\times n}$ the transpose is defined as $A'_{n\times m}$ (in some books $A^T_{n\times m}$) where the rows and columns are interchanged.

$$A_{2\times 4} = \begin{pmatrix} 1 & 4 \\ 3 & -2 \\ 1 & -3 \\ 0 & 1.2 \end{pmatrix} \rightarrow A'_{4\times 2} = \begin{pmatrix} 1 & 3 & 1 & 0 \\ 4 & -2 & -3 & 1.2 \end{pmatrix}$$

- Transposed of a row vector is a column vector and vice versa.

$$X_{3\times 1} = \begin{pmatrix} 1 \\ 5 \\ 4 \end{pmatrix} \rightarrow X'_{1\times 3} = \begin{pmatrix} 1 & 5 & 4 \end{pmatrix}$$

Properties of Transpose Matrix:

- By the definition of transpose matrix we can conclude $(A')' = A$.
- By the definition, $I' = I$. *This property is true for all diagonal matrices.*
- For a square matrix A, if $A' = A$, then A is a symmetric matrix. $\begin{pmatrix} 1 & 0.5 \\ 0.5 & 3 \end{pmatrix}$
- $(kA)' = kA'$

(iii) Multiplication by Constant

- If a matrix is multiplied by a constant and its transpose is taken, then the matrix obtained is equal to transpose of original matrix multiplied by that constant. That is,
- (kA)′ = kA′, where k is a constant

(iv) Multiplication Property of Transpose

- Transpose of the product of two matrices is equal to the product of transpose of the two matrices in reverse order. That is
- (AB)′ = B′A′

$$A = (A^T)^T = \begin{pmatrix} 4 & -1 & 2 \\ 5 & 0 & 3 \end{pmatrix} \quad B = \begin{pmatrix} 2 & -1 & 1 \\ 7 & 5 & -2 \end{pmatrix}$$

$$A + B = \begin{pmatrix} (4+2) & (-1-1) & (2+1) \\ (5+7) & (0+5) & (3-2) \end{pmatrix}$$

$$(A + B)^T = \begin{pmatrix} 6 & -2 & 3 \\ 12 & 5 & 1 \end{pmatrix}$$

$$(A + B)^T = \begin{pmatrix} 6 & 12 \\ -2 & 5 \\ 3 & 1 \end{pmatrix} \text{--------(1)}$$

ADJOINT AND INVERSE OF A MATRIX:

$$[C]=\begin{bmatrix}-61 & 11 & -34\\ -17 & -41 & -26\\ -53 & -29 & -2\end{bmatrix}, \quad adj[A]=\begin{bmatrix}-61 & -17 & -53\\ 11 & -41 & -29\\ -34 & -26 & -2\end{bmatrix}$$

Finding inverse of matrix using adjoint

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Let's learn how to find inverse of matrix using adjoint

But first, let us define adjoint.

For matrix A,

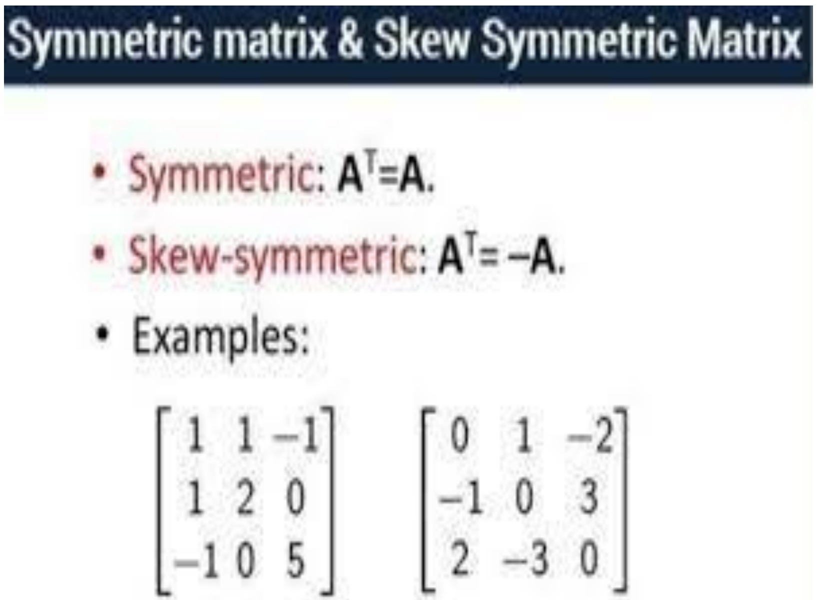
$$A = \begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$$

Adjoint of A is,

$$\text{adj } A = \text{Transpose of} \begin{bmatrix}A_{11} & A_{12} & A_{13}\\ A_{21} & A_{22} & A_{23}\\ A_{31} & A_{32} & A_{33}\end{bmatrix}$$

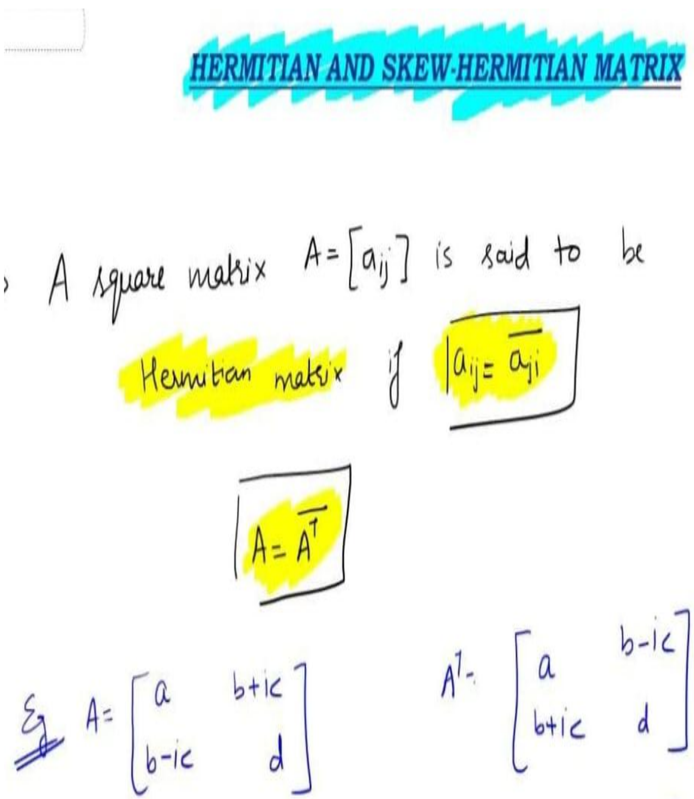
- The adjoint of a matrix (also called the adjugate of a matrix) is defined as the transpose of the cofactor matrix of that particular matrix. On the other hand, the inverse of a matrix A is that matrix which when multiplied by the matrix A give an identity matrix.

SYMMETRIC AND SKEW-SYMMETRIC MATRICES:



- A matrix is symmetric if and only if it is equal to its transpose. All entries above the main diagonal of a symmetric matrix are reflected into equal entries below the diagonal.
  A matrix is skew-symmet ric if and only if it is the opposite of its transpose. All main diagonal entries of a skew-symmetric matrix are zero.

HERMITIAN AND SKEW-HERMITIAN MATRICES:



- **Hermitian matrix:** A square matrix such that a_{ij} is the complex conjugate of a_{ji} for all elements a_{ij} of the matrix i.e. a matrix in which corresponding elements with respect to the diagonal are conjugates of each other. The diagonal elements are always real numbers.

- **Skew-Hermitian matrix:** A square matrix such that

$$a_{ij} = -\overline{a_{ji}}$$

for all elements a_{ij} of the matrix. The diagonal elements are either zeros or pure imaginaries.

Thank You