



# **THIRUTHANGAL NADAR COLLEGE**

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**Selavayal, Chennai-51.**

**A Self-Financing Co-educational College of Arts & Science**

**Affiliated to the University of Madras**

**Accredited with 'B' Grade by NAAC**

**An ISO 9001: 2015 Certified Institution**

**NAME OF THE DEPARTMENT : MATHEMATICS**

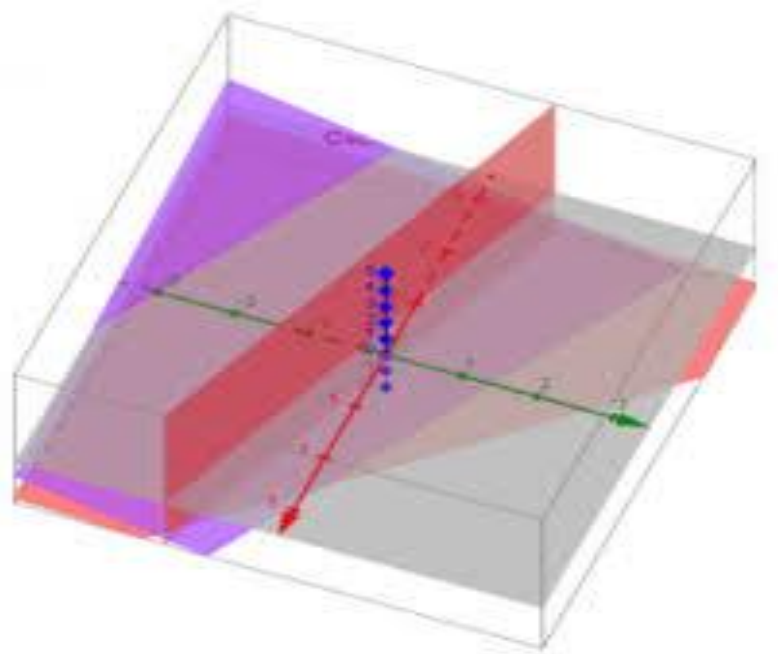
**SUBJECT : ALGEBRA**

**TOPIC : TYPES OF MATRICES**

**STAFF NAME : N.AMUDHA**

# LINEAR ALGEBRA

## MATRIX



# Types Of Matrices

- (1) Real Matrix
- (2) Complex Matrix
- (3) Diagonal Matrix
- (4) Symmetric Matrix
- (5) Upper Triangular Matrix etc.

# Definition of a Matrix

\* Rectangular array of real numbers

m rows by n columns

- \* Named using capital letters
- \* First subscript is row, second subscript is

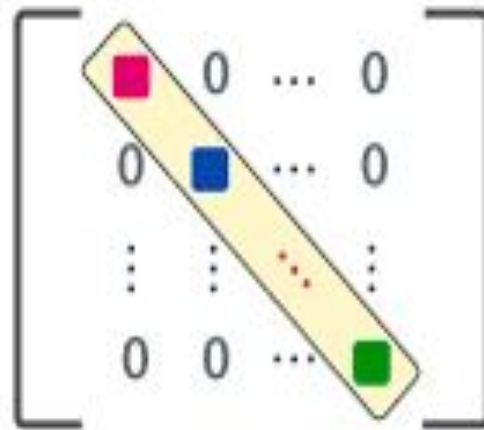
column

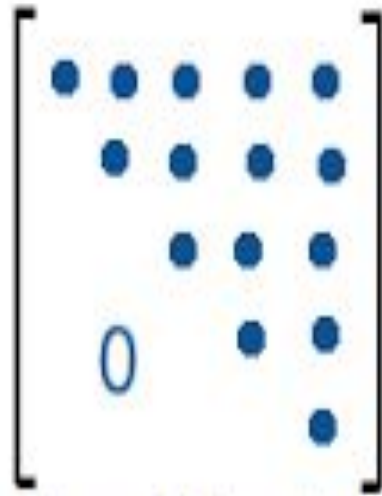
$$\mathbf{A}_{m \times n} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Square Matrix

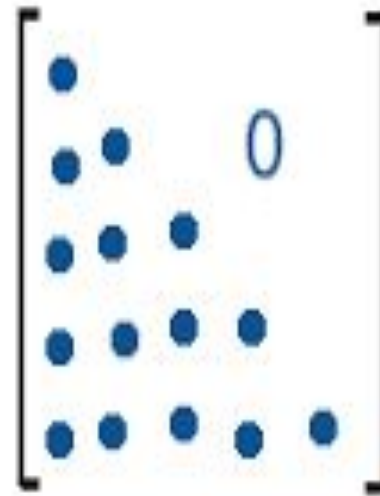


# DIAGONAL MATRIX





Upper Triangular  
Matrix



Lower Triangular  
Matrix

# IDENTITY MATRIX

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

3 x 3



## TYPES OF MATRICES

### A Scalar matrix

A diagonal matrix whose main diagonal elements are equal to the same scalar

A scalar is defined as a single number or constant

$$\begin{bmatrix} a_{ii} & 0 & 0 \\ 0 & a_{ii} & 0 \\ 0 & 0 & a_{ii} \end{bmatrix} \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad \begin{bmatrix} 6 & 0 & 0 & 0 \\ 0 & 6 & 0 & 0 \\ 0 & 0 & 6 & 0 \\ 0 & 0 & 0 & 6 \end{bmatrix}$$

i.e.  $a_{ij} = 0$  for all  $i \neq j$

$a_{ii} = a$  for all  $i = j$

A **Zero Matrix** is a matrix where all elements are 0.

For example:  $\begin{pmatrix} 0 \\ 0 \end{pmatrix}$   $\begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$   $\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$



## 2.3 Symmetric Matrices

### Definition

The **transpose** of a matrix  $A$ , denoted  $A^t$ , is the matrix whose columns are the rows of the given matrix  $A$ .

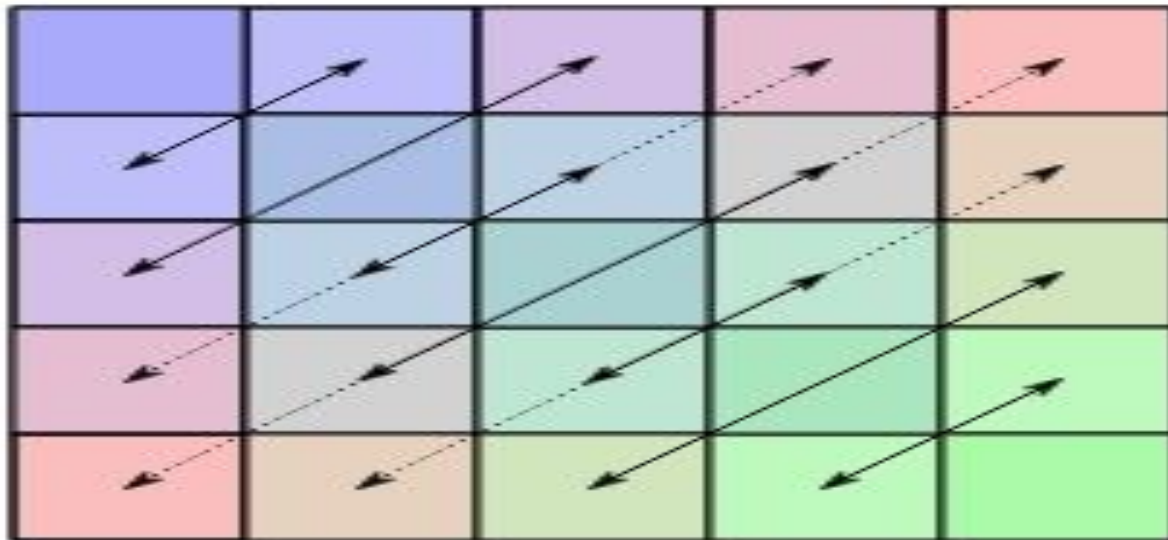
i.e.,  $A : m \times n \Rightarrow A^t : n \times m, (A^t)_{ij} = A_{ji} \quad \forall i, j.$

### Example 15

$$A = \begin{bmatrix} 2 & 7 \\ -8 & 0 \end{bmatrix}, B = \begin{bmatrix} 1 & 2 & -7 \\ 4 & 5 & 6 \end{bmatrix}, \text{ and } C = [-1 \quad 3 \quad 4]$$

$$A^t = \begin{bmatrix} 2 & -8 \\ 7 & 0 \end{bmatrix} \quad B^t = \begin{bmatrix} 1 & 4 \\ 2 & 5 \\ -7 & 6 \end{bmatrix} \quad C^t = \begin{bmatrix} -1 \\ 3 \\ 4 \end{bmatrix}.$$


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# SKEW SYMMETRIC MATRICES



A square matrix  $A$  is said to be a skew symmetric if  $B' = -B$ . And all elements in the principal diagonal of a skew symmetric matrix are zeroes.


$$B = \begin{bmatrix} 0 & e & f \\ -e & 0 & g \\ -f & -g & 0 \end{bmatrix}_{3 \times 3}$$

### DEFINITION

#### Hermitian, Skew-Hermitian, and Unitary Matrices

A square matrix  $\mathbf{A} = [a_{kj}]$  is called

<b>Hermitian</b>	if $\bar{\mathbf{A}}^T = \mathbf{A}$ ,	that is,	$\bar{a}_{kj} = a_{jk}$
<b>skew-Hermitian</b>	if $\bar{\mathbf{A}}^T = -\mathbf{A}$ ,	that is,	$\bar{a}_{kj} = -a_{jk}$
<b>unitary</b>	if $\bar{\mathbf{A}}^T = \mathbf{A}^{-1}$ .		

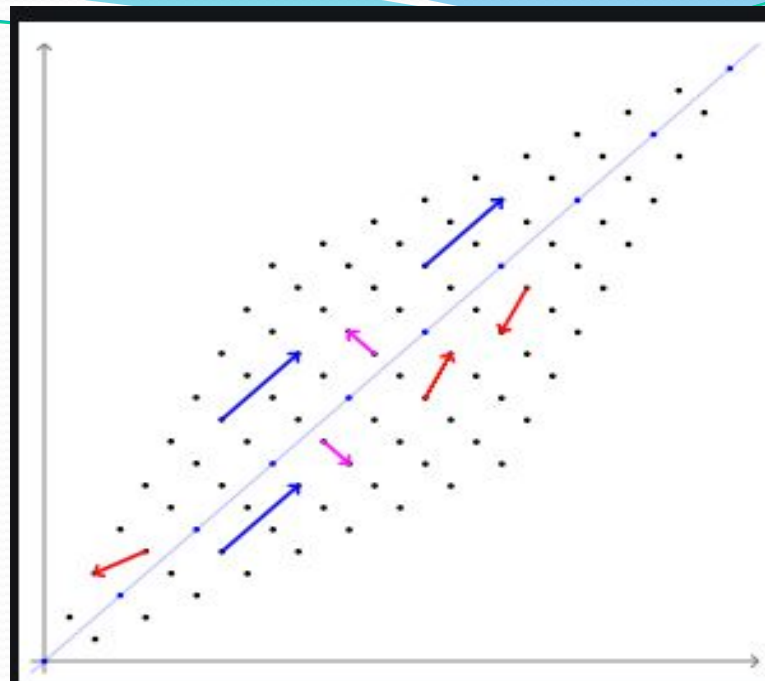
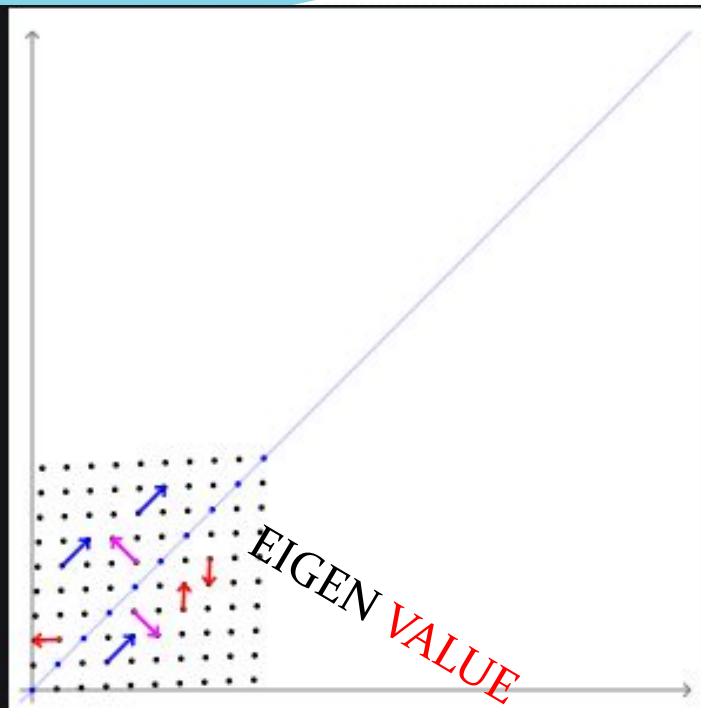
## EIGENVECTORS AND EIGENVALUES

- **Definition:** An **eigenvector** of an  $n \times n$  matrix  $A$  is a nonzero vector  $\mathbf{x}$  such that  $A\mathbf{x} = \lambda\mathbf{x}$  for some scalar  $\lambda$ . A scalar  $\lambda$  is called an **eigenvalue** of  $A$  if there is a nontrivial solution  $\mathbf{x}$  of  $A\mathbf{x} = \lambda\mathbf{x}$ ; such an  $\mathbf{x}$  is called an *eigenvector corresponding to  $\lambda$* .
- $\lambda$  is an eigenvalue of an  $n \times n$  matrix  $A$  if and only if the equation

$$(A - \lambda I)\mathbf{x} = \mathbf{0} \quad \text{----(1)}$$

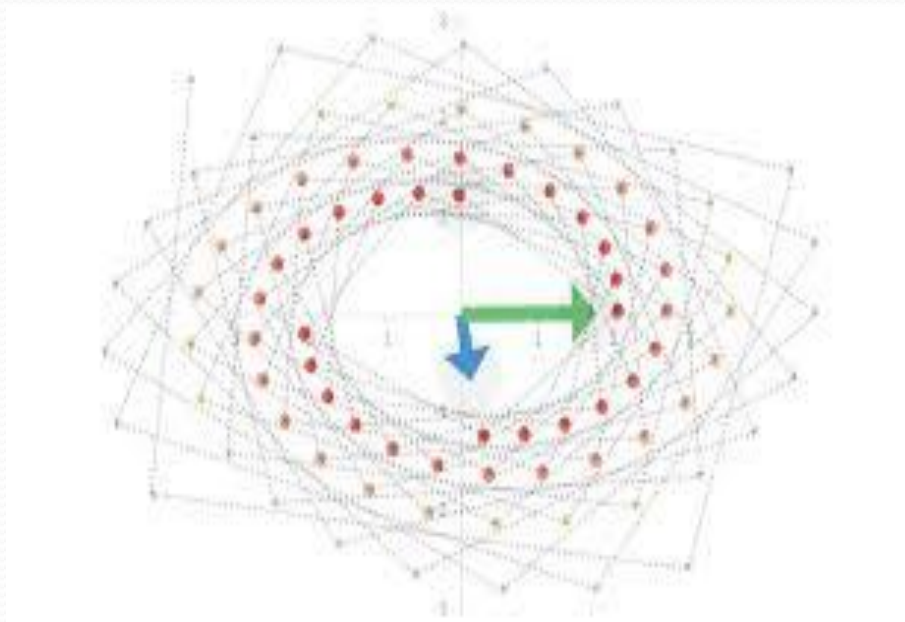
has a nontrivial solution.

- The set of *all* solutions of (1) is just the null space of the matrix  $A - \lambda I$ .



## EIGEN VALUE AND EIGEN VECTOR

## EIGEN VALUE AND EIGEN VECTOR





# LINEAR INDEPENDENCE



- **Definition:** An indexed set of vectors  $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$  in  $\mathbb{R}^n$

is said to be **linearly independent** if the vector equation

$$x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + \dots + x_p\mathbf{v}_p = \mathbf{0}$$

has only the trivial solution. The set  $\{\mathbf{v}_1, \dots, \mathbf{v}_p\}$  is said to be **linearly dependent** if there exist weights  $c_1, \dots, c_p$ , not all zero, such that

$$c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + \dots + c_p\mathbf{v}_p = \mathbf{0} \quad \text{----(1)}$$

## Determining Linear Independence

$$\begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix} \begin{bmatrix} -1 \\ 1 \\ -1 \\ -3 \end{bmatrix} \begin{bmatrix} 2 \\ 5 \\ 9 \\ 6 \end{bmatrix} \xrightarrow{\text{linear combination}} c_1 \begin{bmatrix} 1 \\ 1 \\ 3 \\ 3 \end{bmatrix} + c_2 \begin{bmatrix} -1 \\ 1 \\ -1 \\ -3 \end{bmatrix} + c_3 \begin{bmatrix} 2 \\ 5 \\ 9 \\ 6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 1 & 3/2 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$



$$c_1 = 0$$

$$c_2 = 0$$

$$c_3 = 0$$

then these would be **linearly independent**

Determine if the following functions are dependent or independent.

$$f_1 = \sin(2x), f_2 = \cos(2x)$$

$$W(f_1, f_2) = \begin{vmatrix} \sin(2x) & \cos(2x) \\ 2\cos(2x) & -2\sin(2x) \end{vmatrix}$$

$$W(f_1, f_2) = \begin{vmatrix} f_1 & f_2 \\ f_1' & f_2' \end{vmatrix}$$

	Sledge Hammer	Club Hammer	Sledge Mining Hammer	Copper Sledge Hammer	Ball Pein Hammer	Cross Pein Hammer	Claw Hammer	Soft Face Hammer	Hatchet	Pick
<b>Mining</b>	X	-	X	X	-	X	-	-	X	X
<b>Stone Crushing</b>	X	-	X	-	-	X	-	-	-	-
<b>Demolition</b>	X	-	X	-	-	-	-	-	-	-
<b>Building &amp; Construction</b>	X	X	X	X	-	X	X	-	-	X
<b>Steel Fixing</b>	X	X	X	X	X	-	-	-	-	-
<b>Agriculture</b>	X	X	X	-	-	-	-	-	X	X
<b>Forestry</b>	-	-	-	-	-	-	-	-	X	X
<b>Light Engineering</b>	X	X	X	X	X	-	-	X	-	-

