THIRUTHANGAL NADAR COLLEGE

(Belongs to the Chennaivazh Thiruthangal Hindu Nadar Uravinmurai Dharma Fund)

Selavayal, Chennai-51.

A Self-Financing Co-educational College of Arts & Science

Affiliated to the University of Madras

Accredited with 'B' Grade by NAAC

An ISO 9001: 2015 Certified Institution

AME OF THE DEPARTMENT : MATHEMATICS

SUBJECT : NUMERICAL ANALYSIS I

TOPIC : NUMERICAL METHODS

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NUMERICAL METHODS

Applications :

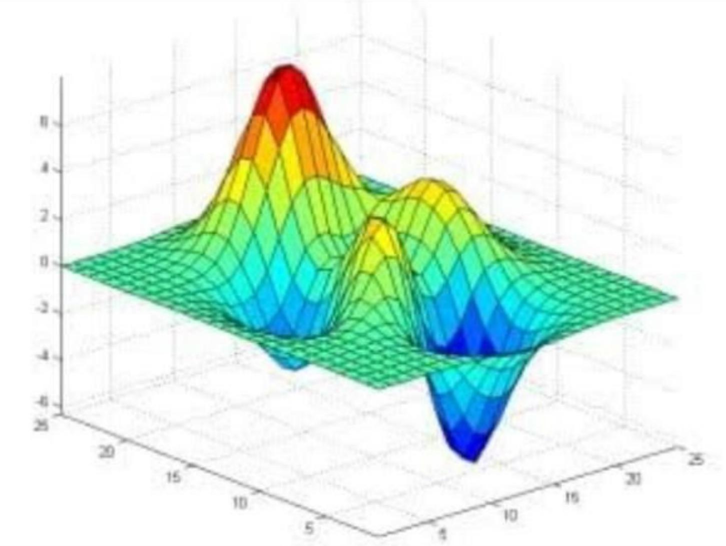
- Engineering
- Scientific computing
- Finding roots
- Heat equation
- Crime detection
- Modeling of airplane
- Estimation of ocean currents
- Weather forecasting

Contents :

- Bisection method
- Method of false position
- Lagrange interpolation
- Gauss interpolation
- Stirling's formula

Introduction :

- Numerical analysis is the study of algorithms that use numerical approximation for the problems of mathematical analysis.
- History of numerical is very long .The first numerical analysis is Babylonian Approximation.



BISECTION METHOD :

- The bisection method is a straightforward technique to find numerical solution of an equation with one unknown
- It is simplest one to solve the transcendental
- It is used to find the roots of a polynomial equation
- The principle is intermediate theorem for continuous functions

Bisection method algorithm:

For any continuous function f(x)

- Find two points ,say a and b such that a<b and $f(a)*f(b)<0$
- Find the midpoint of a and b ,say "t"
- t is the root of the given function if $f(t)=0$; else follow the next step
- Divide the interval [a, b]
- If $f(t)*f(b)<0$,let $a = t$
- Else if $f(t)*f(a)$,let $b = t$
- Repeat above three steps until f(t)

False position method :

False position method is the very old method for solving an equation with one unknown ,in modified form is still in use

Algorithm:

consider the equation f(x)=0 and let x1 ,x2 be two value of x such that f(x1) and f(x2) are of opposite signs .also let x1<x2.the graph of y=f(x) will meet the x-axis at some point between x1 and x2 .

Formula :

$$x_n = x_{n+1} - \frac{f(x_{n+1})}{f(x_{n+2}) - f(x_{n+1})}(x_{n+2} - x_{n+1})$$

Lagrange interpolation :

The Lagrange interpolation is a way to find a polynomial which takes on certain values at arbitray points

Algorithm:

the value of y = f(x) corresponding to any value of x =xi within x0 and xn is called interpolation

Formula :

$$f(x)=f(x_0)\frac{(x-x_1)(x-x_2)\cdots(x-x_n)}{(x_0-x_1)(x_0-x_1)\cdots(x_0-x_n)}+f(x_1)\frac{(x-x_0)(x-x_2)\cdots(x-x_n)}{(x_1-x_0)(x_1-x_2)\cdots(x_1-x_n)}+\dots\dots\dots$$

$$+f(x_n)\frac{(x-x_0)(x-x_1)\cdots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\cdots(x_n-x_{n-1})}$$

This is called Lagrange's interpolation formula and can be used both equal and unequal intervals .

GAUSS INTERPOLATION:

- Interpolation refers to the process of creating new data points given within the given set of data
- The given range of discrete data sets using formula given by gauss
- Combination of gauss forward interpolation and gauss backward interpolation

1. GAUSS FORWARD INTERPOLATION FORMULA

$$P(x) = y_0 + \binom{p}{1}\Delta y_0 + \binom{p}{2}\Delta^2 y_{-1} + \binom{p+1}{3}\Delta^3 y_{-1} + \binom{p+1}{4}\Delta^4 y_{-2} + \binom{p+2}{5}\Delta^5 y_{-2} + \Lambda \text{ where}$$

$$p = \frac{x - x_0}{h} \text{ and } \binom{p}{r} = \frac{p(p-1)(p-2)\cdots(p-r+1)}{r!}$$

OR

$$y_p = y_0 + p\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{(p+1)p(p-1)}{3!}\Delta^3 y_{-1} + \frac{(p+1)p(p-1)(p-2)}{4!}\Delta^4 y_{-2} + \dots$$

.....(A)

2. Gauss Backward interpolation formula

Again starting with Newton's Forward Int. Formula

$$y_0 + p\Delta y_0 + \frac{p(p-1)}{\lfloor 2}\Delta^2 y_0 + \frac{p(p-1)(p-2)}{\lfloor 3}\Delta^3 y_0 + \dots + \frac{p(p-1)\dots[p-(n-1)]}{\lfloor n}\Delta^n y_0 \quad \dots(1)$$

we have

$$\Delta y_0 - \Delta y_{-1} = \Delta^2 y_{-1} \quad \Rightarrow \quad \Delta y_0 = \Delta^2 y_{-1} + \Delta y_{-1}$$

similarly

$$\Delta^2 y_0 = \Delta^3 y_{-1} + \Delta^2 y_{-1} \quad and \quad \Delta^3 y_0 = \Delta^4 y_{-1} + \Delta^3 y_{-1}$$

Stirling's formula :

Stirling's Formulae

Statement: If $\ldots, x_{-2}, x_{-1}, x_0, x_1, x_2, \ldots$ are given set of observations with common difference h and let $\ldots, y_{-2}, y_{-1}, y_0, y_1, y_2, \ldots$ are their corresponding values, where $y = f(x)$ be the given function then

$$y_p = y_0 + p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} + \ldots \text{ where } p = \frac{x - x_0}{h}$$

Proof: Stirling's Formula will be obtained by taking the averageardodifferenGauss fo formula and Gauss Backward difference formula.

We know that, from Gauss forward difference formula

$$y_p = y_0 + p\,\Delta y_0 + \frac{p(p-1)}{2!}\Delta^2 y_{-1} + \frac{p(p-1)(p+1)}{3!}\Delta^3 y_{-1} + \frac{p(p-1)(p+1)(p-2)}{4!}\Delta^4 y_{-2} + \ldots \quad \text{----} > (1)$$

Also, from Gauss backward difference formula

$$y_p = y_0 + p\,\Delta y_{-1} + \frac{p(p+1)}{2!}\Delta^2 y_{-1} + \frac{p(p+1)(p-1)}{3!}\Delta^3 y_{-2} + \frac{p(p+1)(p-1)(p+2)}{4!}\Delta^4 y_{-2} + \ldots \quad \text{----} > (2)$$

Now, $Stirling's\ Formula = \frac{1}{2}(Gauss\ forward\ formula + Gauss\ backward\ formula)$

$$\therefore\ y_p = y_0 + p\left(\frac{\Delta y_0 + \Delta y_{-1}}{2}\right) + \frac{p^2}{2!}\Delta^2 y_{-1} + \frac{p(p^2-1)}{3!}\left(\frac{\Delta^3 y_{-1} + \Delta^3 y_{-2}}{2}\right) + \frac{p^2(p^2-1)}{4!}\Delta^4 y_{-2} + \ldots$$


Thank You