



THIRUTHANGAL NADAR COLLEGE

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Selavayal, Chennai-51.

A Self-Financing Co-educational College of Arts & Science

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NAME OF THE DEPARTMENT : MATHEMATICS

SUBJECT : OPERATIONS RESEARCH

TOPIC : TRANSPORTATION PROBLEM

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Transportation Problem

What is transportation problem?

- **The transportation problem is a special type of linear programming problem where the objective is to minimize the cost of distributing a product from a number of sources or origins to a number of destinations.**
- **Because of its special structure, the usual simplex method is not suitable for solving transportation problems. These problems require a special method of solution.**

AIM of Transportation Problem:

- ❑ **To find out the optimum transportation schedule keeping in mind cost of transportation to be minimized.**
- ❑ **The origin of a transportation problem is the location from which shipments are dispatched.**
- ❑ **The destination of a transportation problem is the location to which shipments are transported.**
- ❑ **The unit transportation cost is the cost of transportation one unit of the consignment from an origin to a destination.**

Objectives of Transportation problem

Determination of a transportation plan of a single commodity from a number of sources , from a number of destinations such that total cost of transportation is minimized.

Application of Transportation problem

- It is used to compute transportation routes in such a way as to minimize transportation cost for finding out locations of warehouses.
- It is used to find out locations of transportation corporations depots where insignificant total cost difference may not matter.
- Minimize shipping costs from factories to warehouses (or from warehouses to retail outlets).
- Determine lowest cost location for new factory, warehouse, office, or other outlets facility.
- Find minimum cost production schedule that satisfies firms demand and production limitations.

Two Types of transportation problem:

□ **Balanced Transportation problem:**

Where the total supply equally total demand.

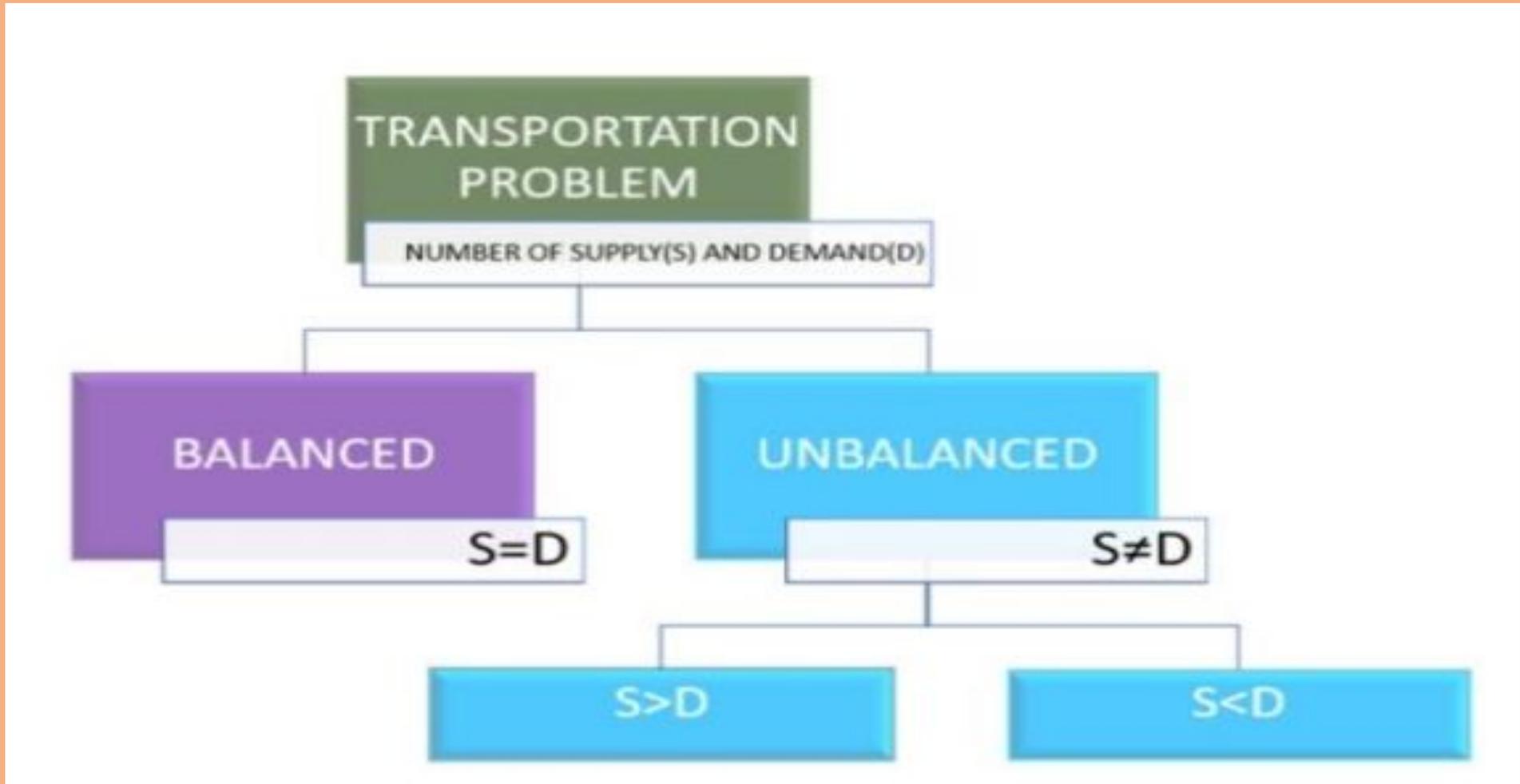
□ **Unbalanced transportation Problem:**

Where the total supply is not equally total demand.

Phases of solution of transportation problem:

- **Phases I : Obtains the Initial Basic Feasible Solution**
- **Phases II : Obtains the optimal Basic solution.**

Types of transportation problem:



Solution to the transportation problem:

Initial Basic Feasible Solution:

- 1. North – West corner Rule(NWCR)**
- 2. Least Cost method (LCM)**
- 3. Vogle Approximation method (VAM)**

Optimum Basic solution :

- 1. Modified Distribution Method (MODI method)**
- 2. Stepping stone method.**

North- West Corner Rule

Definition:

The North-West Corner Rule is a method adopted to compute the Initial feasible solution of the transportation problem.

The name North-West corner is given to this method because the basic variables are selected from the extreme left corner.

It is most systematic and easiest method for obtaining Initial Feasible Basic solution.

Steps of North- West Corner Rule

Step 1:

Select the upper left (North-West) cell of the transportation matrix and allocate minimum of supply and demand. Or $\min(A_1, B_1)$ Value in the cell.

Step 2:

If $A_1 < B_1$, then allocation made is equal to the supply available at the first source (A_1 is first row), then move vertically down to the cell (2,1)

If $A_1 > B_1$, then allocation made is equal to demand of the first destination (B_1 in first column), then move horizontally to the cell (1,2)

If $A_1 = B_1$, then allocation the value of A_1 or B_1 and then move to cell (2,2).

Step 3:

Continue the process until an allocation is made in the Southeast corner cell of the transportation table.

Least Cost method

Definition:

The Least Cost Method is another method used to obtain the initial feasible solution for the transportation problem. Here, the allocation begins with the cell which has the minimum cost. The lower cost cells are chosen over the higher-cost cell with the objective to have the least cost of transportation.

The Least Cost Method is considered to produce more optimal results than the North-west Corner because it considers the shipping cost while making the allocation, whereas the North-West corner method only considers the availability and supply requirement and allocation begin with the extreme left corner, irrespective of the shipping cost.

Vogel Approximation method (VAM method)

Definition:

The Vogel's Approximation Method or VAM is an iterative procedure calculated to find out the initial feasible solution of the transportation problem. Like Least cost Method, here also the shipping cost is taken into consideration, but in a relative sense.

The Vogel Approximation method is an improved version of the minimum cell cost method and the North West corner method that in general produces better initial Basic Feasible Solution, that reports a smaller value in the objective (minimization) function of a balanced Transportation problem. (Sum of the supply = Sum of the demand).

MODI Method (Modified Distribution Method)

- ❑ Determine an initial Basic Feasible Solution using any one of the three methods given below.
 - North-West corner method
 - Least Cost method
 - Vogel Approximation method

- ❑ Determine the value of dual variables, U_i and V_j , using $U_i + V_j = C_{ij}$

- ❑ Compute the opportunity cost using $C_{ij} - (U_i + V_j)$.

MODI Method (Modified Distribution Method)

- ❑ Check the sign of each opportunity cost, if the opportunity cost of all the unoccupied cells are either positive or zero, then given solution is the optimal solution on the other hand. If one or more unoccupied cells has negative opportunity cost. The given solution is not an optimal solution and further savings in transportation cost are possible.
- ❑ Select the unoccupied cells with the smallest negative opportunity cost as the cell to be included in the next solution.
- ❑ Draw a closed path or loop for the unoccupied cells selected in the preview step. Please note that the right angle turn in this path is permitted only at occupied cells and at the original unoccupied cells
- ❑ Assign alternate plus and minus signs at the unoccupied cells on the corner point of the closed path with a plus sign at the being evaluated.

MODI Method (Modified Distribution Method)

- ❑ Determine the maximum number of units that should be shipped to this unoccupied cell.
- ❑ The smallest value with a negative position on the closed path indicates the number of units that can be shipped to the entering cell.
- ❑ Now, add this quantity to all the cells on the corner points of the closed path marked with plus signs, and Subtract it from those cells marked with minus signs.
- ❑ In this way, an unoccupied cell becomes an occupied cell.

Stepping Stone Method

Definition:

The Stepping Stone Method is used to check the optimality of the initial feasible solution determined by using any of the method Viz. North-West Corner, Least Cost Method or Vogel's Approximation Method. Thus, the stepping stone method is a procedure for finding the potential of any non-basic variables (empty cells) in terms of the objective function.

Through Stepping stone method, we determine that what effect on the transportation cost would be in case one unit is assigned to the empty cell. With the help of this method, we come to know whether the solution is optimal or not.

The series of steps are involved in checking the optimality of the initial feasible solution using the stepping stone method:

- 1. The prerequisite condition to solve for the optimality is to ensure that the number of occupied cells is exactly equal to $m+n-1$, where 'm' is the number of rows, while 'n' is equal to the number of columns.**
- 2. Firstly, the empty cell is selected and then the closed path is created which starts from the unoccupied cell and returns to the same unoccupied cell, called as a "closed loop". For creating a closed loop the following conditions should be kept in mind:**

The series of steps are involved in checking the optimality of the initial feasible solution using the stepping stone method:

In a closed loop, cells are selected in a sequence such that one cell is unused/unoccupied, and all other cells are used/occupied.

A pair of Consecutive used cells lies either in the same row or the same column.

No three consecutive occupied cells can either be in the same row or column.

The first and last cells in the closed loop lies either in the same row or column.

Only horizontal and vertical movement is allowed.

Once the loop is created, assign “+” or “-” sign alternatively on each corner cell of the loop, but begin with the “+” sign for the unoccupied cell.

The series of steps are involved in checking the optimality of the initial feasible solution using the stepping stone method:

4. Repeat these steps again until all the unoccupied cells get evaluated.
5. Now, if all the computed changes are positive or are equal to or greater than zero, then the optimal solution has been reached.
6. But in case, if any, value comes to be negative, then there is a scope to reduce the transportation cost further.
Then, select that unoccupied cell which has the most negative change and assign as many units as possible. Subtract the unit that added to the unoccupied cell from the other cells with a negative sign in a loop, to balance the demand and supply requirements.

THANK YOU ...!