THIRUTHANGAL NADAR COLLEGE

(Belongs to the Chennaivazh Thiruthangal Hindu Nadar Uravinmurai Dharma Fund)

Selavayal, Chennai-51.

A Self-Financing Co-educational College of Arts & Science

Affiliated to the University of Madras

Accredited with 'B' Grade by NAAC

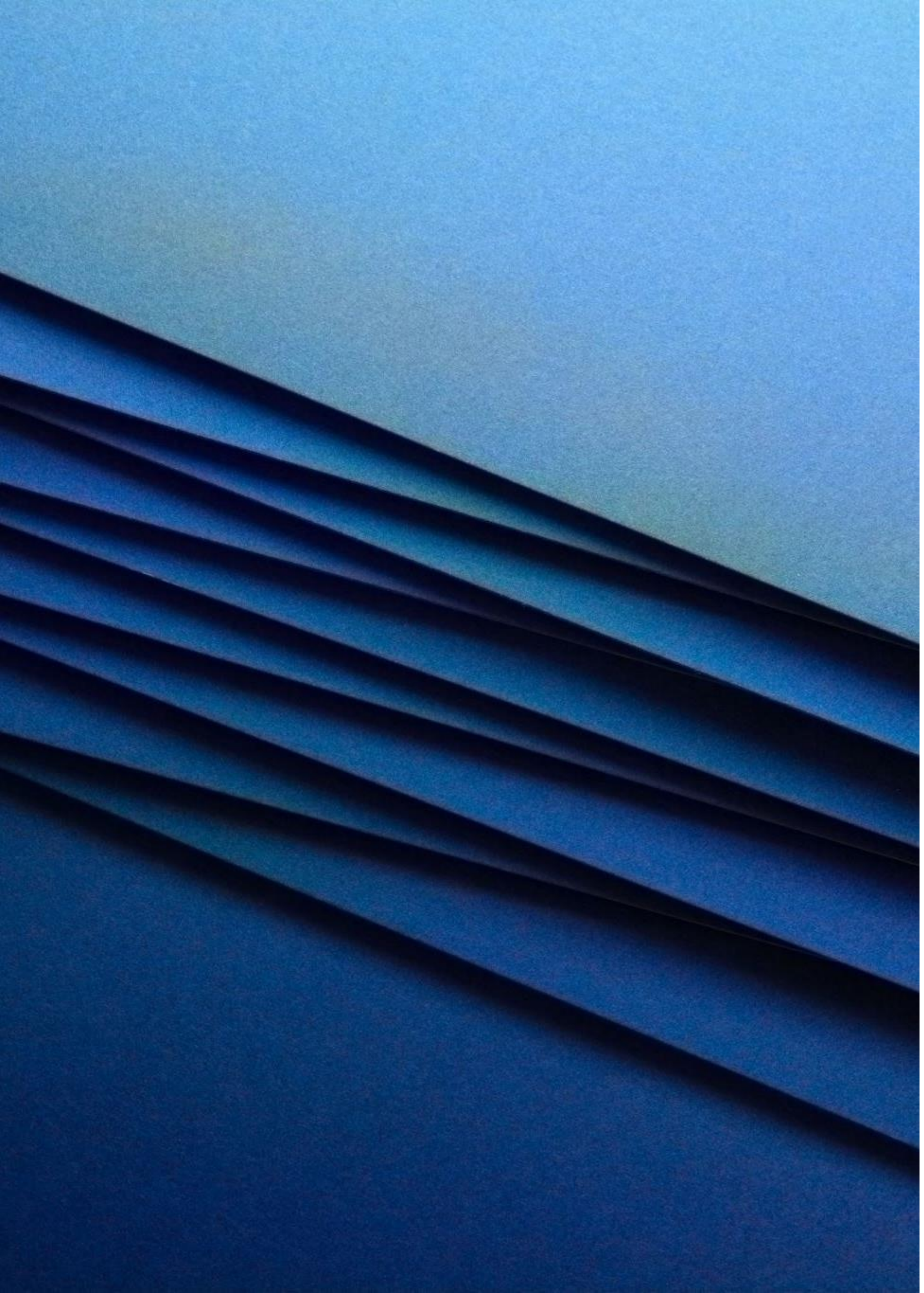
An ISO 9001: 2015 Certified Institution

NAME OF THE DEPARTMENT : MATHEMATICS

SUBJECT : STATICS

TOPIC : PROBLEMS BASED ON LAMI'S THEOREM

STAFF NAME : CHITHAMBARA BHARATHY S



STATICS

S is circumcentre of ΔABC. If force of magnitudes P,Q,R acting along SA,SB,SC are in equilibrium. Show that P,Q,R are in the ratio.

1) $\frac{P}{sin2A} = \frac{Q}{sin2B} = \frac{R}{sin2C}$

2) $\frac{P}{a^2(b^2+c^2-a^2)} = \frac{Q}{b^2(c^2+a^2-b^2)} = \frac{R}{c^2(a^2+b^2-c^2)}$

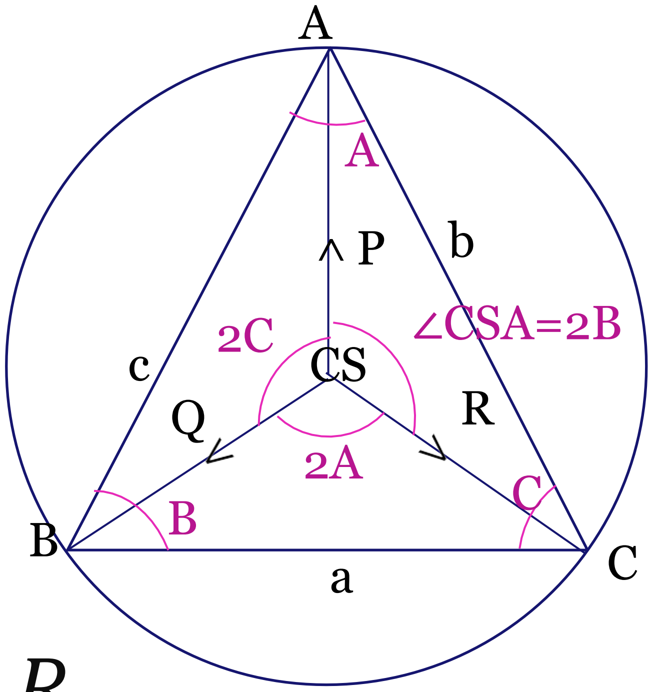
3) $\frac{P}{\Delta BSC} = \frac{Q}{\Delta CSA} = \frac{R}{\Delta ASB}$

SOLUTION:

GIVEN:

The forces P,Q,R acting at O

and are in equilibrium



LAMI'S THEOREM:

$$\frac{P}{\sin\angle BSC}=\frac{Q}{\sin\angle CSA}=\frac{R}{\sin\angle ASB} \rightarrow \boxed{*}$$

i) S is circumference of ΔABC

A is a point on circumference of circumcircle.

$\therefore \angle BAC = A;\ \angle BSC = 2A; \angle CSA = 2B; \angle ASC = 2C.$

$$\boxed{*} \longrightarrow \frac{P}{\sin 2A} = \frac{Q}{\sin 2B} = \frac{R}{\sin 2C} \longrightarrow \boxed{i}$$

ii) $$\boxed{i} \longrightarrow \frac{P}{2\sin A \cos A} = \frac{Q}{2\sin B \cos B} = \frac{R}{2\sin C \cos C}$$

W.K.T,

Sine formula:

$$\frac{a}{sinA} = \frac{b}{sinB} = \frac{c}{sinC}$$

sinA:sinB:sinC=a:b:c

Cosine formula:

$$cosA = \frac{b^2 + c^2 - a^2}{2bc}$$

$$cosB = \frac{c^2 + a^2 - b^2}{2ac}$$

$$cosC = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\frac{P}{a\left(\frac{a}{a}\right)\left(\frac{b^2+c^2-a^2}{2bc}\right)}=\frac{Q}{b\left(\frac{b}{b}\right)\left(\frac{c^2+a^2-b^2}{2ac}\right)}=\frac{R}{c\left(\frac{c}{c}\right)\left(\frac{a^2+b^2-c^2}{2ab}\right)}$$

$$\frac{P}{a^2\,\frac{(b^2+c^2-a^2)}{2abc}}=\frac{Q}{b^2\,\frac{(c^2+a^2-b^2)}{2abc}}=\frac{R}{c^2\,\frac{(a^2+b^2-c^2)}{2abc}}$$

$$\Rightarrow \frac{P}{a^2(b^2+c^2-a^2)} = \frac{Q}{b^2(c^2+a^2-b^2)} = \frac{R}{c^2(a^2+b^2-c^2)} \rightarrow \boxed{\text{ii}}$$

iii) $\boxed{\text{i}} \rightarrow \frac{P}{sin2A} = \frac{Q}{sin2B} = \frac{R}{sin2C}$

if S is circumference of ABC. Then

Radius R=SA=SB=SC

Area, $\Delta BSC = (1/2)\ SB.SC\ Sin2A$

$\Delta BSC = (1/2)\ R.R\ Sin2A$

$Sin2A = \frac{2}{R^2} \Delta BSC$

III^{ly} $\sin 2B = \frac{2}{R^2} \Delta CSA$; $Sin2c = \frac{2}{R^2} \Delta ASB$

i → $$\frac{P}{\frac{2}{R2}\Delta BSC} = \frac{Q}{\frac{2}{R2}\Delta CSA} \quad \frac{R}{R2\,\Delta ASB}$$

$$\Rightarrow \frac{P}{\Delta BSC} = \frac{Q}{\Delta CSA} = \frac{R}{\Delta ASB}$$ → iii

Hence proved.

THANK YOU